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Analysis and Calculation of Concentric Coil Area

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Abstract. In disc magneto electric generator, the output voltage is proportional to the coil surrounding area. Establish the sectorial coil model, trapezoidal coil model and rectangular coil model and build simplified calculation formulaused for calculation of coil surrounding area. The maximum error between simplified and the precisecalculation formula is 6% that can meet the needs of engineering design. It can be used to guide structural design and optimization of this type of generator.

Introduction

Disc magneto electric generator is an ideal power device with advantages as light weight, small size, compact structure, small moment of inertia, etc. According to different connection methods, the coil can be divided into lap winding, wave winding and concentric winding as shown in Fig. 1. For disc generator, the lap winding structure is more complex, particularly in regard to the adjacent coils overlapping each other. Structure of concentric coil is simple, therefore easy processing and high yield[1-3].

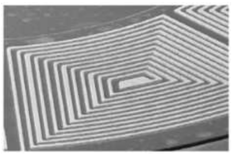


Fig. 1 Imperial College London developed a micro-generator with concentric coil

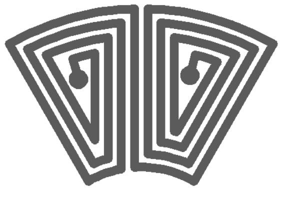


Fig. 2 Sketch of sector unit

According to Faraday's law of electromagnetic induction, the output voltage within the sector (shown in Fig. 2), U (t) can be expressed as

$$U(t) = -\frac{d\emptyset_C(t)}{dt} = -S_Z\frac{dB_\delta(t)}{dt} \tag{1}$$

B_δ means static magnetic induction between the gap. S_Z means total area surrounded by concentric coils in each sector. Φ_C means magnetic flux within each sector.Eq1.shows that S_Z is a critical parameter affecting the output voltage. In the design of disc magneto electric generator, calculation of S_Z is required. Usually modeling is done in CAD and different models are required for eachindependent design which is complicated and not conducive to the coil design optimization analysis. Establishing sectorial coil model, trapezoidal coil model and rectangular coil model, surrounding area of coil is calculated accurately with error less than 6%. The method both reduces the difficulty of analysis and makes the design easier.

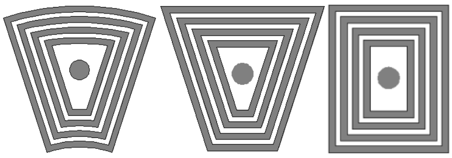
Modeling of Sectorial Coil

Simplify the coil model shown in Fig. 2 as follows,

1) Approximate continuous coil to discrete form, as shown in Fig. 3(a)

2)Approximate sectorial coil model to trapezoidal coil model and rectangular coil model, as shown in Fig.3(b)(c)

In actual generator, the number of the magnetic poles generally can reach 10~30. Number of magnetic poles in disc generator is less, commonly 3~10. This paper studies magnetic poles of 6~10.



(a) Sectorial coil model (b) trapezoidal coil model (c) rectangular coil model

Fig. 3 Modeling of coil

Define total area surrounded by concentric sectorial coil within a sector is S_{ss}, The structure and parameters are shown in Fig.4.

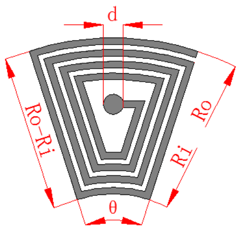


Fig. 4 Structure and parameters of coil

Definition of parameters in Fig.4: R_o is the Outer radius of the sectorial coil; R_i is the inner radius of the sectorial coil; θ is the angle between the foul lines of the sector; d is the diameter of the center hole; w is the distance between wire lines.

Next, verify the rationality of the approximate coil model. When the number of magnetic poles, p, is 6~10, corresponding θ is 30°~18°, Sectorial coil model is shown in Fig.5.

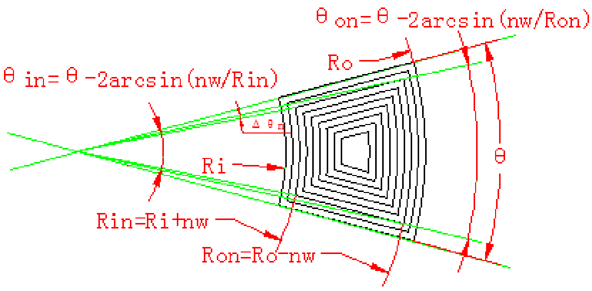


Fig. 5 Sectorial coil model

Definition of parameters in Fig.5: In circle n, R_{on} is the outer radius; R_{in} is the inner radius; θ_{on} is angle between the outer arc; θ_{in} is angle between the inner arc; $\Delta\theta n$ is half of difference betweenθ_{on} and θ_{in}.

There are N_t circles in sectorial coil model, area of circle n, S_{sn},can be expressed as,

$$S_{sn} = \frac{1}{2}\theta_{on}\left(R_{on}{}^{2} - R_{in}{}^{2}\right) - 2[\frac{1}{2}\Delta R_{in}(R_{on} - R_{in})] \quad (2)$$

$$\begin{cases} R_{on} = R_o - nw \\ R_{in} = R_i - nw \end{cases} \quad (3)$$

$$\begin{cases} \theta_{on} = \theta - 2\arcsin(nw/R_{on}) \\ \theta_{in} = \theta - 2\arcsin(nw/R_{in}) \\ \Delta\theta_n = 0.5(\theta_{on} - \theta_{in}) \end{cases} \quad (4)$$

Total area of sectorial coil model S_{ss}

$$S_{ss} = \sum_{n=0}^{N_t-1} S_{sn} \quad (5)$$

With Eq.2~Eq.5 ,When R_o=5mm,R_i=3mm,w=0.01mm,N_t=10, calculation area surrounded by coil is 16.4170mm^2 and accurate area modeled by CAD is 16.4216mm^2 which means the error is 0.03%. Eq.2~Eq.5 is rational for accurate calculation. Based on the above analysis, sectorial coil model is reasonable for calculation of total area S_{SS} surrounded by coil.

Modeling of trapezoidal coil model and rectangular coil model

Formulation of sectorial coil is accurate while the derivation and expression is complex that is not good for analysis of variable in it. So trapezoidal coil model and rectangular coil model is put forward, as shown in Fig. 6 and Fig. 7.

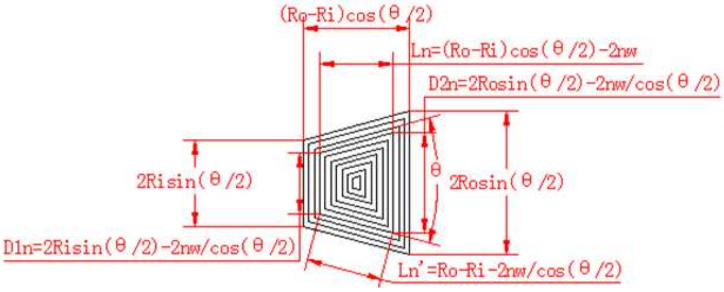


Fig. 6 Trapezoidal coil model

Definition of parameters in Fig. 6: In circle n, D_{1n} is the topline; D_{2n} is the baseline; L_n' is the hypotenuse and L_n is the height.

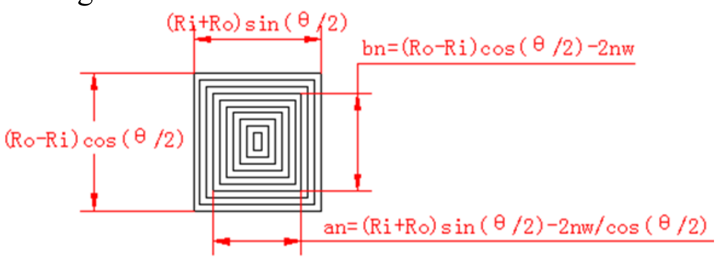


Fig. 7 Rectangular coil model

Definition of parameters in Fig.7: In circle n, a_n is the shorter edge and b_n is the longer edge.

The area expression of the trapezoidal coil model and the rectangular coil model is the same, the formula is as follows

There are N_t circles in coil model, area of circle nS_{tn} can be expressed as,

$$S_{tn} = \left[(R_o - R_i)\cos\left(\frac{\theta}{2}\right) - 2nw\right]\left[(R_o + R_i)\sin\left(\frac{\theta}{2}\right) - \frac{2nw}{\cos(\theta/2)}\right] \quad (6)$$

Total area of trapezoidal coil model or rectangular coil model S_{ts},

$$S_{ts} = \sum_{n=0}^{N_t-1} S_{tn} \quad (7)$$

Analysis based on rectangular coil model

Expression of rectangular coil model is simple and easy for analysis. Rectangular coil model and parameters are shown in Fig. 8.

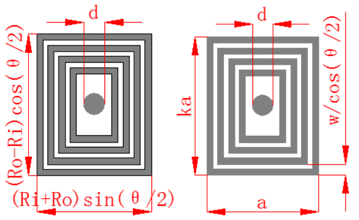


Fig. 8 (a)Original parameters (b) Simplified parameters

Definition of parameters in Fig. 8(b): a is the shorter edge and ka is the longer edge of the outermost circle. Deriving Formula:

$$S_{jn} = [ka - 2nw]\left[a - 2n\frac{w}{\cos(\theta/2)}\right] \quad (8)$$

$$N_t = \left[\frac{a-d}{2w/\cos(\theta/2)}\right] \quad (9)$$

$$S_{js} = \sum_{n=0}^{N_t-1} S_{jn} \quad (10)$$

When the number of magnetic poles, p, is 6~10, corresponding θ is 30°~18°; Ro>Ri. So $\cos(0.5\theta)^{-1}$ is 1.012~1.035 and approximate to 1 for calculation. So the formula above can be expressed as,

$$S_{jn} = (ka - 2nw)(a - 2nw) \quad (11)$$

$$N_t = \left[\frac{a-d}{2w}\right] \quad (12)$$

$$S_{js} = \sum_{n=0}^{N_t-1} S_{jn} = (a^2 - d^2)\left[0.5 + \frac{(k-1)a}{2(a+d)} + \frac{d^2}{6w(a+d)} - \frac{d(3k-1)}{12w} + \frac{(a+d)(3k-1)}{12w} + \frac{w}{3(a+d)}\right] \quad (13)$$

Further simplify the rectangular coil model that central through-hole diameter d can be ignored. Substitute d=0mm into Eq.13,

$$S_{js} = ka^2\left(\frac{1}{2} + \frac{w}{3ak} + \frac{a}{4w} - \frac{a}{12wk}\right) \quad (14)$$

$$a = (R_o + R_i)\sin\left(\frac{\theta}{2}\right) \quad (15)$$

$$k = \frac{(R_o - R_i)\cos(\theta/2)}{(R_o + R_i)\sin(\theta/2)} \tag{16}$$

$$\frac{dk}{dR_o} = \frac{2R_i\cos(\pi/2p)}{\sin(\pi/2p)(R_i+R_o)^2} > 0 \tag{17}$$

$$\frac{dk}{dR_o} = \frac{-2R_o\cos(\pi/2p)}{\sin(\pi/2p)(R_i+R_o)^2} < 0 \tag{18}$$

$$\theta = \frac{\pi}{p} \tag{19}$$

From Eq. 17 and Eq. 18, enhance the value of R_o and decrease the value of R_i is conducive to improve the output voltage which means the wiring area $A=\pi(R_{co}^2-R_{ci}^2)$ is larger.

Take the sectorial coil model as a standard to calculate the error of trapezoidal coil model and rectangular coil model, as shown in Table1.

Table 1.Accuracy of trapezoidal coil model and rectangular coil model

No.	Number of poles	Size[mm]	Sector	Trapezoid		Rectangle	
			Area [mm^2]	Area [mm^2]	Error [%]	Area [mm^2]	Error [%]
1	θ=30° p=6	R_o=5; R_i=3; w=0.1; N_t=10	16.4	15.2	7.6	15.2	7.6
2		R_o=5; R_i=3; w=0.02; N_t=30	68.1	63.3	6.9	63.3	6.9
3		R_o=5.5;R_i=1.5; w=0.1; N_t=10	32.0	29.5	7.8	29.5	7.8
4	θ=18° p=10	R_o=5;R_i=3; w=0.1;N_t=6	7.5	7.3	2.6	7.3	2.6
5		R_o=5; R_i=3; w=0.02;N_t=30	32.4	31.5	2.9	31.5	2.9
6		R_o=5.5;R_i=1.5; w=0.1;N_t=6	13.3	12.9	2.7	12.9	2.7

In Table 1, calculation area by trapezoidal coil model and rectangular coil model is the same. Compare data 1 and 2, when the shape is the same, the less the distance between lines, the more the number of rings, the error is larger. Compare data 1 and 3, 4 and 6, error decreases when the size enhances. Compare 1,2 and 3, as well as 4,5 and 6, when the number of magnetic poles, p, is certain, the error is Relatively constant. Error falls as the number of magnetic poles rises. The lager p is, the smaller θ is and the number of rings contained with the concentric coil is less. The maximum error of Eq.14 is 6% that it is reasonable for approximate calculation of coil area.

Conclusion

Performance and processing characteristics of disc permanent magnet generator makesstudies on concentric winding area is necessary.In this paper, based on geometric forms mathematical model can be divided into sectorial coil model, trapezoidal coil model and rectangular coil model. Among them, sectorial coil model is used for precise calculation and reference standard for trapezoidal and rectangular coil model. Trapezoidal coil model is a transition between sectorial and rectangular coil model. The rectangular coil model with a simple expression is used for analysis of effects of each parameter on output voltage and Eq. 14 can be used for accurate calculation of coil area. Not only reduces the difficulty of the coil design, three models are applied to qualitative analysis and precise calculation that makes design more clear.

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